STABILITY ALLOWANCES OF ELASTICOVISCOPLASTIC BODIES

A. N. Sporykhin and V. G. Trofimov UDC 539.374

On the basis of three-dimensional linearized stability equations, the straining process of elasticoviscoplastic bodies is investigated when they are compressed along the x_3 axis by forces of intensity p and along the x_1 and x_2 axes by forces of intensity q. Precritical strains are small and homogeneous. Stability of slabs is investigated as an example. A graphical dependence of the critical loads on the properties and geometry of the slabs is presented.

1. In [1, 2] the general solutions of static and dynamic stability equations [3, 4] are presented for hardening elasticoviscoplastic bodies when they are compressed along the x_3 axis.

The solutions presented here, analogously to the results obtained for elasticoviscoelastic and plastic bodies [5, 6] in the case of small homogeneous precritical strains, allow us to investigate a broad class of stability problems of elasticoviscoplastic bodies.

The stress–strain state of a three-dimensional body up to the loss of stability is given by the relationships

$$\sigma_{11}{}^0 = \sigma_{22}{}^0 = -q, \quad \sigma_{33}{}^0 = -p, \quad \sigma_{ij}{}^0 = 0 \quad (i \neq j)$$
$$2e_{11}{}^{p_0} = 2e_{22}{}^{p_0} = -e_{33}{}^{p_0} = 2(p - q - k\sqrt{1.5})/3c, \quad e_{ij}{}^{p_0} = 0 \quad (i \neq j) \tag{1.1}$$

The linearized stability equations [3, 4] are represented in the form

$$L_{ij}u_j = 0 \qquad (i, j = 1, 2, 3) \tag{1.2}$$

where the differential operators have the form

$$L_{ij} = (\lambda + \mu + a_j b_i)\frac{\partial^2}{\partial x_i \partial x_j} + \delta_{ij}\Big[(\mu - q)\frac{\partial^2}{\partial x_1{}^2} + (\mu - q)\frac{\partial^2}{\partial x_2{}^2} + (\mu - p)\frac{\partial^2}{\partial x_3{}^2} - \rho s^2\Big] \quad (b_i = s_{ii}{}^0 - ce_{ii}{}^{p_0}, \quad a_j = 4\mu^2(b_1 + b_2 + b_3 - 3b_j)\,[3k^2(2\mu + c + s\eta)]^{-1})$$

For a cylindrical body with a curvilinear contour of the cross section the general solution of stability equations has the form

$$u_n = \frac{\partial}{\partial \tau}\Psi_1 - \frac{\partial^2}{\partial n\,\partial x_3}\Psi, \quad u_\tau = -\frac{\partial}{\partial n}\Psi_1 - \frac{\partial^2}{\partial \tau\,\partial x_3}\Psi$$
$$u_3 = \frac{\lambda + 2\mu + a_1 b_1 - q}{\lambda + \mu + a_3 b_1}\left(\Delta + \frac{\mu - p}{\lambda + 2\mu + a_1 b_1 - q}\frac{\partial^2}{\partial x_3{}^2} - \frac{\rho s^2}{\lambda + 2\mu + a_1 b_1 - q}\right)\Psi \tag{1.3}$$

Here by n and τ we have denoted respectively the normal and the tangent to the contour of the cross section.

The functions Ψ and Ψ_1 are determined from the equations

Voronezh. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 144–147, July–August, 1973. Original article submitted March 5, 1973.

$$\left(\Delta + \frac{\mu - p}{\mu - q}\frac{\partial^2}{\partial x_3^2} - \frac{\rho s^2}{\mu - q}\right)\Psi_1 = 0 \qquad \left(\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right)$$

$$\left[\left(\Delta + \frac{\mu - p}{\lambda + 2\mu + a_1 b_1 - q}\frac{\partial^2}{\partial x_3^2} - \frac{\rho s^2}{\lambda + 2\mu + a_1 b_1 - q}\right)\left(\Delta + \frac{\lambda + 2\mu + a_3 b_3 - p}{\mu - q}\frac{\partial^2}{\partial x_3^2} - \frac{\rho s^2}{\mu - q}\right) - \right.$$
$$\left. - \frac{(\lambda + \mu + a_3 b_1)(\lambda + \mu + a_1 b_3)}{(\lambda + 2\mu + a_1 b_1 - q)(\mu - q)}\Delta\frac{\partial^2}{\partial x_3^2}\right]\Psi = 0 \tag{1.4}$$

In the static formulation (s = 0) the functions Ψ_i are the solutions of the equations

$$\left(\Delta + \xi_i^2 \frac{\partial^2}{\partial x_3^2}\right)\Psi_i = 0 \qquad (\Psi = \Psi_2 + \Psi_3) \tag{1.5}$$

where the constants ξ_i^2 have the form

$$\xi_1^2 = \frac{\mu - p}{\mu - q}, \quad \xi_{2,3}^2 = A \pm \left[A^2 - \frac{(\lambda + 2\mu + a_3 b_3 - p)(\mu - p)}{(\lambda + 2\mu + a_1 b_1 - q)(\mu - q)}\right]^{1/2}$$
$$A = \frac{1}{2}\left[\frac{\lambda + 2\mu + a_3 b_3 - p}{\mu - q} + \frac{\mu - p}{\lambda + 2\mu + a_1 b_1 - q} - \frac{(\lambda + \mu + a_3 b_1)(\lambda + \mu + a_1 b_3)}{(\lambda + 2\mu + a_1 b_1 - q)(\mu - q)}\right]$$

In the case of plane strain ($x_1 x_3$) the solution of the system of equations (1.2) can be represented in the form

$$u_1 = L_{33}\Psi, \qquad u_3 = -L_{31}\Psi \tag{1.6}$$

The solution of Eq. (1.4), which is periodic with respect to the x_3 axis, is written in the form

$$\Psi = (C_m^1 e^{k_2 x_1} + C_m^2 e^{-k_2 x_1} + C_m^3 e^{k_3 x_1} + C_m^4 e^{-k_3 x_1})\sin(\gamma x_3)$$
$$k_{2,3}^2 = D \pm \sqrt{F}, \quad D = A\gamma^2 + \frac{\rho s^2(\lambda + 3\mu + a_1 b_1 - 2q)}{2(\mu - q)(\lambda + 2\mu + a_1 b_1 - q)}$$
$$F = D^2 - \frac{(\mu - p)(\lambda + 2\mu + a_3 b_3 - p)\gamma^4 - \rho s^2(\lambda + 3\mu + a_3 b_3 - 2p)\gamma^2 + \rho^2 s^4}{(\mu - q)(\lambda + 2\mu + a_1 b_1 - q)} \tag{1.7}$$
$$(\gamma = \pi m / l; \qquad m = 1, 2, 3, \ldots, \infty)$$

The solution (1.7) satisfies the conditions of pin-jointed support at the ends in an integral sense.

2. We shall investigate the stability of straining of a slab of thickness 2h and length l. It is assumed that up to the instant of the loss of stability the stress–strain state of the slab is described by the relationships (1.1), while at the instant of the loss of stability, strain occurs in the $x_1 x_3$ plane.

The boundary conditions on the side surface $x_1 = \pm h$ lead to the relationships

$$\left(\frac{\partial^2}{\partial x_1^2} + B_1\frac{\partial^2}{\partial x_3^2} - s^2 B_3\right)\frac{\partial \Psi}{\partial x_1} = 0, \quad \left(\frac{\partial^2}{\partial x_1^2} - B_2\frac{\partial^2}{\partial x_3^2} + s^2 B_4\right)\frac{\partial \Psi}{\partial x_3} = 0$$

$$B_1 = \frac{\lambda + 2\mu + a_3 b_3 - p}{\mu - q} - \frac{(\lambda + \mu + a_1 b_3)(\lambda + a_3 b_1)}{(\mu - q)(\lambda + 2\mu + a_1 b_1 - r)}, \quad B_3 = \frac{\rho}{\mu - q}$$
$$B_2 = \frac{\mu(\lambda + 2\mu + a_3 b_3 - p)}{(\mu - r)(\lambda + a_1 b_3) - \mu(\mu - q)}, \tag{2.1}$$
$$B_4 = \frac{\rho\mu}{(\mu - r)(\lambda + \mu + a_1 b_1) - \mu(\mu - q)}$$

In the expressions (2.1) we must put r = q if the load q is a "dead" load; we must put r = 0 if the load is a "following" load.

From Eqs. (1.7) and (2.1), with the condition that nonzero solutions exist, we obtain the transcendental equations for the determination of the critical loads

$$\frac{k_2(k_2^2 - B_1\gamma^2 - s^2 B_3)(k_3^2 + B_2\gamma^2 + s^2 B_4)}{k_3(k_3^2 - B_1\gamma^2 - s^2 B_3)(k_2^2 + B_2\gamma^2 + s^2 B_4)} = \operatorname{th}(k_3 h)\operatorname{cth}(k_2 h) \tag{2.2}$$

$$\frac{k_2(k_2^2 - B_1\gamma^2 - s^2 B_3)(k_3^2 + B_2\gamma^2 + s^2 B_4)}{k_3(k_3^2 - B_1\gamma^2 - s^2 B_3)(k_2^2 + B_2\gamma^2 + s^2 B_4)} = \operatorname{th}(k_2 h)\operatorname{cth}(k_3 h) \tag{2.3}$$

The relationships (2.2) and (2.3) are stability criteria respectively when the displacement u_1 is even and odd with respect to x_1.

For a thin plate the expansion of the trigonometric multipliers in a power series and neglecting the compressibility of the material, in the case of a following load q, from (2.2) we obtain the algebraic equation

$$[3(1-q)+(3+2q)\alpha^2]\rho s^2-\gamma_1^2\{3(1-q)(p+q)-\\-\alpha^2[3p+d-4-q(1-p-q-d)]\}=0,\quad \gamma_1=\pi/l,\\ \alpha=\gamma_1 h,\ d=6/(2+c+s\eta) \qquad (2.4)$$

Here all quantities having the dimensions of stress are referred to the quantity E/3.

We rewrite (2.4) in the form

$$d_3s^3+d_2s^2+d_1s+d_0=0\\ d_0=(2+c)d_1/\eta+6\alpha^2\gamma_1^2(1+q),\ d_1=\eta\gamma_1^2[3p(1+\alpha^2)-\\-4\alpha^2+pq(\alpha^2-3)-q(3q+q\alpha^2+\alpha^2)],\\ d_2=\rho(2+C)[3(1-q)+(3+2q)\alpha^2],\ d_3=\eta d_2(2+c)^{-1} \qquad (2.5)$$

An application of the Hurwitz criterion [7] shows that instability arises only if $d_0 = 0$. Here the characteristic index s passes into the right half-plane of the complex variable via s = 0. For the critical load p we have

$$p=\frac{2\alpha^2+2q[3q+\alpha^2(q-2)]+c[3q^2+\alpha^2(4+q+q^2)]}{3(2+c)(1+\alpha^2)+q(\alpha^2-3)} \qquad (2.6)$$

In the case of a dead load a static type of instability is possible [4]. For a thin plate from (2.2) we can obtain the expression of the critical force p with accuracy up to α^4

$$p=\frac{q}{q-1}+\frac{\alpha^2}{3(q-1)^3}[(1-q)d-(2-q)^2]+\frac{\alpha^4}{3(q-1)^5}\Big\{\frac{1}{3}(2q-3)[(1-q)d-(2-\\-q)^2]-{}^2/_5[(q-1)(2-q-d)-q][(1-q)d-(q-2)^2]\Big\} \qquad (2.7)$$

We shall investigate the effect of the properties of the material and the geometrical dimensions of the slab on the mode of buckling and on the magnitude of the critical force. We assume that only a uniform compressive force p (q = 0) acts on the slab. As was shown, two modes of buckling are possible: lateral (2.2) and barrel-shaped (2.3). Equations (2.2) and (2.3) in the static formulation were solved numerically on a BÉSM-4 computer for various values of the parameters ν, c_1, and $2h/l$; ν is Poisson's ratio, $c_1 = c/E$.

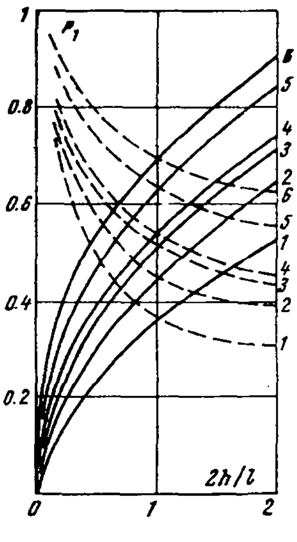

Fig. 1

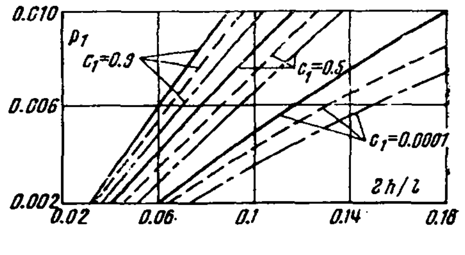

Fig. 2

The dependence of the magnitude of the critical load $p_1 = p/E$ on the mode of buckling and the parameters indicated above is presented in Fig. 1. The solid lines correspond to the lateral buckling, while the dashed lines correspond to the barrel-shaped buckling. The curves with numbers 1, 2, 3 and 4, 5, 6 correspond to the hardening coefficients $c_1 = 0.001$ and 0.75. The curves with numbers 1,4 correspond to $\nu = 0.2$; 2.5 correspond to $\nu = 0.3$; 3,6 correspond to $\nu = 0.5$, $c_1 = 0.0001$ and 0.75. As is seen from Fig. 1, in the case of small precritical strains no barrel-shaped buckling, in view of the limit critical loads, is observed. The lateral buckling takes place for plates with the ratio $2h/l < 0.2$ (Fig. 2). The solid lines correspond to $\nu = 0.5$, the dashed lines correspond to $\nu = 0.3$, and the dash-dotted lines correspond to $\nu = 0.2$. For $2h/l > 0.2$ the strength properties of the slabs are decisive.

LITERATURE CITED

1. A. N. Sporykhin and V. G. Trofimov, "On constructing the general solution of linearized equations of the plane stability problem of elasticoviscoplastic bodies," Tr. Nauchn.-Issled. In-ta Matem. Voronezhsk. Un-ta, No. 4 (1971).
2. A. N. Sporykhin and V. G. Trofimov, "Stability of elasticoviscoplastic bodies," Prikl. Mekhan., 8, No. 9 (1972).
3. V. V. Novozhilov, Foundations of the Nonlinear Theory of Elasticity [in Russian], Gostekhizdat, Moscow (1948).
4. V. V. Bolotin, Nonconservative Problems of the Theory of Elastic Stability [in Russian], Fizmatgiz, Moscow (1961).
5. A. N. Guz', "On a three-dimensional theory of stability of deformation of a material with rheological properties," Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, No. 6 (1970).
6. A. N. Guz', Stability of Three-Dimensional Deformable Bodies [in Russian], Naukova Dumka, Kiev (1971).
7. B. P. Demidovich, Lectures on the Mathematical Theory of Stability [in Russian], Nauka, Moscow (1967).